Incremental Harmonic Balance Method for Piecewise Structural Nonlinear Aeroelastic System

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Abstract: The incremental harmonic balance method is extended to the analysis of the periodic response for an aeroelastic system with piecewise structural nonlinearity. The process of the incremental harmonic balance method is derived for such a system. The nonlinear terms are studied to convert the nonlinear aeroelastic equation to a set of algebraic equations, which can provide an idea for solution of other nonlinearities. In order to accelerate the convergence of the solution to algebraic equations, the dominant frequency component in the response is achieved by the fast Fourier transform of the numerical solution, which can avoid the blind assumption of the nonlinear aeroelastic system solution. Finally, the solution of the aeroelastic response with piecewise structural nonlinearity via the incremental harmonic balance method is achieved. The results are in good agreement with numerical results obtained by the Runge-Kutta method. The comparison verifies the correctness of the proposed method. The effect of the numbers of harmonics on the solution precision, as well as the effect of the free-play and stiffness ratio on the response amplitude, is discussed. The incremental harmonic balance method is very effective for a piecewise structural nonlinear aeroelastic system, and its application is expanded.

1. Introduction

In the aeronautical field, there are inevitably a variety of nonlinear factors between components of the aircraft wings, such as free-play nonlinearity and cubic nonlinearity, resulting in complex aeroelastic response, including limit cycle oscillation, bifurcation, chaos etc.[1] Therefore, it is of great significance to perform structural nonlinear flutter analysis.

Woolston et al.[2] and Shen et al.[3] first used the harmonic balance method to analyze the nonlinear aeroelastic response of the structure, and then it was widely used. The classical harmonic balance method (that is, the assumptive solution contains only one major harmonic in the analysis) can also be called the description function method, and cannot predict the high-order harmonic response.[4,5] In the case of nonlinear aeroelastic analysis with cubic nonlinearity of a two-dimensional airfoil, it is found that a greater number of harmonics can be used to describe the quadratic bifurcation, but, with the increase of the number of harmonics, the nonlinear term derivation becomes too complicated.[6] Liu et al.[7] derived the higher-order harmonic balance (HOHB) method to investigate the high harmonics of the airfoil motions with a free-play control surface, in which two additional variables were introduced to result in more equations and more complex solution techniques. Dimitriadis et al.[8] devised a HOHB method to extend the effectiveness of the harmonic balance method to systems undergoing secondary Hopf bifurcations, in which the proposed harmonic shifting technique can allow the HOHB method to accurately estimate both branches of limit cycles occurring after the second bifurcation. As the computational power available to researchers has increased, so has the order of calculated harmonic balance solutions. However, the computational cost of a harmonic balance solution increases with the

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square of the order. Additionally, the solution of the nonlinear algebraic problem at the heart of the harmonic balance relies on a good initial guess for the unknown coefficients. If there are many such coefficients the probability that a good guess will be available is very low, and the harmonic balance scheme may well fail. Vio et al.[9] introduced a search procedure using genetic algorithms to evaluate the coefficients of a harmonic balance solution, showing that genetic algorithms could provide high-quality initial guesses for the harmonic balance coefficients. Dai et al.[10] utilized the harmonic balance method to obtain the periodic solutions for a two-dimensional airfoil with cubic nonlinearity in pitch undergoing subsonic flow, in which the Jacobian matrix was explicitly derived to improve the accuracy and efficiency of the harmonic balance method.

However, the incremental harmonic balance method,[11] combining the incremental method and the harmonic balance method in the numerical calculation, is also a semi-analytical, semi-numerical method proposed to avoid the direct solution of complicated harmonic equilibrium equations.[12,13] Lau analyzed the dynamic characteristics of a piecewise linear stiffness system, and extended it to an asymmetric piecewise linear stiffness system.[14,15] Cai Ming et al.[16] applied the incremental harmonic balance method to the cubic nonlinearity in both plunge and pitch motion of the two-dimensional airfoil aeroelastic system, and analyzed limit cycle oscillation as well as the effect of the number of harmonics on the response. Liu et al.[17] investigated the aeroelasticity of an airfoil with hysteresis nonlinearity in the pitching degree-of-freedom by the incremental harmonic balance method, in which an undetermined coefficient method was extended to cope with the problem of expanding the hysteresis nonlinearity. Xiong et al.[18] improved the incremental harmonic balance in order to determine periodic solutions of bilinear hysteretic systems, in which the so-called two-point tracing algorithm was introduced, resulting in more complex solution techniques.

To the best of our knowledge, application of harmonic balance method and incremental harmonic balance method are based on a two-dimensional airfoil, and the aerodynamic force is also quasi-steady. The nonlinear aeroelastic system focuses on the cubic nonlinear system or the piecewise linear stiffness system. The final order of the aeroelastic equation is low. It is easy to apply the incremental harmonic balance method to such nonlinear aeroelastic analysis. Moreover, for the piecewise linear stiffness system, the additional calculation is necessary, implying that there is more difficulty to solve. However, due to coupling of unsteady aerodynamic and structural response, the quasi-steady aerodynamic model is used, in which only the aerodynamic stiffness term is proportional to the pitch degree of freedom. The results are poorly representative. Therefore, we propose a more effective approach for a more complex nonlinear aeroelastic system.

In this paper, a two-dimensional airfoil aeroelastic model based on the unsteady vortex method presented in Ref. [19] is taken as an example, along with a piecewise structural nonlinearity consisting of cubic nonlinearity and free-play nonlinearity (i.e., a special case of piecewise linear stiffness system). The incremental harmonic balance method is applied to the nonlinear aeroelastic analysis, and the approximate solution of the response is obtained. First, the nonlinear aeroelastic equation of the two-dimensional airfoil is rewritten in the state space, and then the incremental harmonic balance procedure is deduced. The processing method of the piecewise nonlinear term is investigated, and the explicit expression form is achieved. Meanwhile, a more concise iterative calculation process is demonstrated. Finally, compared with the numerical solutions obtained by the Runge-Kutta method, the influences of the number of harmonics on the solution precision and of the free play and stiffness ratio on the response amplitude are analyzed.

2 Wing Model and Dynamic Equations

The two-dimensional airfoil is shown in Fig. 1, and the parameters are given in Ref. [19]. In order to conveniently apply the incremental harmonic balance method, the nonlinear aeroelastic equation of the two-dimensional airfoil is written as follows:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{D}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} + \mathbf{F}(\mathbf{X}) = \mathbf{0} \tag{1}$$

where $\mathbf{M}=\begin{bmatrix}\bar{\mathbf{M}}+\bar{\mathbf{A}}_{c33} & \mathbf{0}\\ \bar{\mathbf{A}}_{c13} & \mathbf{0}\end{bmatrix}$, $\mathbf{D}=\begin{bmatrix}U\bar{\mathbf{A}}_{c32}-U\bar{\mathbf{B}}_{c33} & \bar{\mathbf{A}}_{c33}\\ U\bar{\mathbf{A}}_{c12}-U\bar{\mathbf{B}}_{c13} & \mathbf{I}_{R\times R}\end{bmatrix}$, $\mathbf{K}=\begin{bmatrix}-\bar{\mathbf{K}}+U^2\bar{\mathbf{B}}_{c32} & -U\bar{\mathbf{B}}_{c31}\\ \mathbf{0} & -(U/\Delta\bar{x})\mathbf{\Lambda}_R\end{bmatrix}$, $\bar{\mathbf{K}}=\begin{bmatrix}\left(\bar{\eta}_\omega^2/\bar{r}_\alpha^2\right) & 0\\ 0 & 0\end{bmatrix}$, $\mathbf{X}=\begin{bmatrix}\mathbf{u}^T & \mathbf{q}^T\end{bmatrix}^T$, and $\mathbf{u}=\begin{bmatrix}h & \alpha\end{bmatrix}^T$. h refers to the plunge degree of freedom, and α refers to the pitch degree of freedom. $\mathbf{q}=\begin{bmatrix}q_1 & \cdots & q_m & q_{m+1} & q_{m+2} & \cdots & q_{m+n}\end{bmatrix}^T$, which is the generalized coordinate vector associated with aerodynamic force. $\mathbf{\Lambda}=blockdiag(-\lambda_{(1)},-\lambda_{(2)},...,-\lambda_{(m)},\mathbf{\Omega}_{(1)},\mathbf{\Omega}_{(2)},...,\mathbf{\Omega}_{(n)})$, where m and n are the number of real eigenvalues and conjugate complex eigenvalues of the aerodynamic system, respectively, and R is the number of reserved aerodynamic modes. $\mathbf{F}(\mathbf{X})=[0 \quad \bar{f}_a(\alpha)+\bar{f}_b(\alpha)+\bar{f}_c(\alpha) \quad \mathbf{0}^T]^T$, where $\bar{f}_a(\alpha)=\begin{cases}\alpha,\\ 0,\\ \alpha,\end{cases}$ $\bar{f}_b(\alpha)=\begin{cases}-\alpha_s,\\ 0,\\ \alpha_s,\end{cases}$ $\bar{f}_c(\alpha)=\begin{cases}\eta_k(\alpha-\alpha_s)^3, \alpha>\alpha_s\\ 0, -\alpha_s\le\alpha\le\alpha_s\\ \eta_k(\alpha+\alpha_s)^3, \alpha\le-\alpha_s\end{cases}$, α_s is the free-play, and η_k is the stiffness ratio, which represents the ratio of the nonlinear stiffness coefficient to the linear stiffness coefficient. The relevant elements in the matrix are given in Ref. [19].

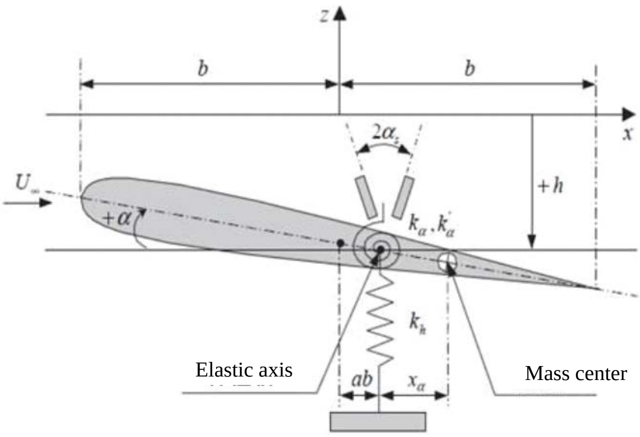


Fig. 1 Typical section of a two-dimensional airfoil [19]

3 Approximate Solution Based on Incremental Harmonic Balance Method

ω is set to the vibration frequency of the system, and $\tau=\omega t$. The first step of the incremental harmonic balance method is the incremental Newton-Raphson process. ω_0 and $\mathbf{X}_0$ are set to a certain state in the vibration, and the neighboring states can be expressed in incremental form:

$$\mathbf{X}=\mathbf{X}_0+\Delta\mathbf{X}, \quad \omega=\omega_0+\Delta\omega \tag{2}$$

Substituting Eq. (2) into Eq. (1), omitting the higher-order term, and expressing $\mathbf{F}(\mathbf{X})$ by the first-order Taylor series approximation yields the incremental equation:

$$\omega_0^2\mathbf{M}\Delta\mathbf{X}''+\omega_0\mathbf{D}\Delta\mathbf{X}'+\mathbf{K}\Delta\mathbf{X}+\mathbf{F}'(\mathbf{X}_0)\Delta\mathbf{X}=\bar{\mathbf{R}}-(2\omega_0\mathbf{M}\mathbf{X}_0''+\mathbf{D}\mathbf{X}_0)\Delta\omega \tag{3}$$

where """ and "'" represent the second and first derivatives with respect to time, respectively. $\bar{\mathbf{R}}=-(\omega_0^2\mathbf{M}\mathbf{X}_0^2+\omega_0\mathbf{D}\mathbf{X}_0+\mathbf{K}\mathbf{X}_0)-\mathbf{F}(\mathbf{X}_0)$, which is the error vector. When ω_0 and $\mathbf{X}_0$ are the exact solution of Eq. (1), the error vector $\bar{\mathbf{R}}=\mathbf{0}$.

The second step of the incremental harmonic balance method is the harmonic balance procedure. The approximate solution of Eq. (3) can be expressed as follows:

$$x_{j0}=\sum_{k=1}^{N_c}a_{jk}\cos(k-1)\tau+\sum_{k=1}^{N_s}b_{jk}\sin k\tau=\mathbf{C}_s\mathbf{A}_j \tag{4}$$

$$\Delta x_{j0}=\sum_{k=1}^{N_c}\Delta a_{jk}\cos(k-1)\tau+\sum_{k=1}^{N_s}\Delta b_{jk}\sin k\tau=\mathbf{C}_s\Delta\mathbf{A}_j \tag{5}$$

where $\mathbf{C}_s=[1\quad\cos\tau\quad\cos 2\tau\quad\cdots\quad\cos(N_c-1)\tau\quad\sin\tau\quad\sin 2\tau\quad\cdots\quad\sin N_s\tau]$, and $\mathbf{A}_j=\begin{bmatrix}a_{j1} & a_{j2} & \cdots & a_{jN_c} & b_{j1} & b_{j2} & \cdots & b_{jN_s}\end{bmatrix}$, $j=1,2,\cdots 2+R$.

Equations (4) and (5) are written in matrix form:

$$\mathbf{X}_0=\mathbf{S}\mathbf{A}\text{ , }\Delta\mathbf{X}=\mathbf{S}\Delta\mathbf{A} \tag{6}$$

where $\mathbf{S}=diag(\underbrace{\mathbf{C}_s\quad\cdots\quad\mathbf{C}_s}_{2+R})$, $\mathbf{A}=\begin{bmatrix}\mathbf{A}_1^T & \cdots & \mathbf{A}_{2+R}^T\end{bmatrix}^T$.

Substituting Eq. (6) into Eq. (3), and applying the Galerkin averaging process, which is the harmonic balance process, one can obtain

$$\begin{aligned}&\int_0^{2\pi}\delta(\Delta\mathbf{X})^T(\omega_0^2\mathbf{M}\Delta\mathbf{X}''+\omega_0\mathbf{D}\Delta\mathbf{X}'+\mathbf{K}\Delta\mathbf{X}+\mathbf{F}'(\mathbf{X}_0)\Delta\mathbf{X})d\tau\\&=\int_0^{2\pi}\delta(\Delta\mathbf{X})^T(\bar{\mathbf{R}}-(2\omega_0\mathbf{M}\mathbf{X}_0''+\mathbf{D}\mathbf{X}_0)\Delta\omega)d\tau.\end{aligned} \tag{7}$$

Finally, the above equations are integrated and simplified to obtain the algebraic equations by $\Delta\mathbf{A}$ and $\Delta\omega$, which are expressed as

$$\mathbf{K}_{mc}\Delta\mathbf{A}=\mathbf{R}+\mathbf{R}_{mc}\Delta\omega \tag{8}$$

where $\mathbf{K}_{mc}=\omega_0^2\bar{\mathbf{M}}+\omega_0\bar{\mathbf{D}}+\bar{\mathbf{K}}+\mathbf{K}^{NL}$, $\mathbf{R}=-(\omega_0^2\bar{\mathbf{M}}+\omega_0\bar{\mathbf{D}}+\bar{\mathbf{K}})\mathbf{A}-\mathbf{R}^{NL}$, $\mathbf{R}_{mc}=-(2\omega_0\bar{\mathbf{M}}+\bar{\mathbf{D}})\mathbf{A}$, $\bar{\mathbf{M}}=\int_0^{2\pi}\mathbf{S}^T\mathbf{M}\mathbf{S}''d\tau$, $\bar{\mathbf{D}}=\int_0^{2\pi}\mathbf{S}^T\mathbf{D}\mathbf{S}'d\tau$, and $\mathbf{K}=\int_0^{2\pi}\mathbf{S}^T\mathbf{K}\mathbf{S}d\tau$. $\mathbf{K}^{NL}$ and $\mathbf{R}^{NL}$ are the first-order Taylor series approximations of the nonlinear forces caused by $\mathbf{F}'(\mathbf{X}_0)$ and $\mathbf{F}(\mathbf{X}_0)$. In this study, the nonlinear force is only related to the pitch degree of freedom. Therefore, the processing of the elements related to the pitch degree of freedom in $\mathbf{K}^{NL}$ and $\mathbf{R}^{NL}$ is worked out here, and the remaining elements are zero. In this case, the nonlinear force of the pitch degree of freedom in $\mathbf{F}(\mathbf{X})$ is assumed to be $f(\alpha)=\overline{f}_a(\alpha)+\overline{f}_b(\alpha)+\overline{f}_c(\alpha)$, and the explicit form of $\mathbf{K}^{NL}$ and $\mathbf{R}^{NL}$ can be worked out as follows:

$$\begin{cases}\mathbf{K}_\alpha^{NLa}=\sum_{m=0}^{M}\int_{\theta_m}^{\theta_{m+1}}\mathbf{C}_{stmp}^T H(u_m)(\operatorname{sgn}a)\mathbf{C}_{stmp}d\theta\\ \mathbf{K}_\alpha^{NLc}=\sum_{m=0}^{M}\int_{\theta_m}^{\theta_{m+1}}3R_a\mathbf{C}_{stmp}^T H(u_m)(\mathbf{C}_{stmp}\mathbf{A}_2+\operatorname{sgn}c)^2\mathbf{C}_{stmp}d\theta\\ \mathbf{R}_\alpha^{NLa}=\sum_{m=0}^{M}\int_{\theta_m}^{\theta_{m+1}}\mathbf{C}_{stmp}^T H(u_m)(\mathbf{C}_{stmp}\mathbf{A}_2\operatorname{sgn}a)d\theta\\ \mathbf{R}_\alpha^{NLb}=\sum_{m=0}^{M}\int_{\theta_m}^{\theta_{m+1}}\mathbf{C}_{stmp}^T H(u_m)(\operatorname{sgn}b)d\theta\\ \mathbf{R}_\alpha^{NLc}=\sum_{m=0}^{M}\int_{\theta_m}^{\theta_{m+1}}R_a\mathbf{C}_{stmp}^T H(u_m)(\mathbf{C}_{stmp}\mathbf{A}_2+\operatorname{sgn}c)^3d\theta\end{cases} \tag{9}$$

where $\mathbf{C}_{stmp}=[1\quad\cos\theta\quad\cos 2\theta\quad\cdots\quad\cos(N_c-1)\theta\quad\sin\theta\quad\sin 2\theta\quad\cdots\quad\sin N_s\theta]$, and M represents the number of solutions of the equation $|\alpha(\tau)|=\alpha_s$ in the interval $(0,2\pi)$. Here, $\theta_0=0$, $\theta_{M+1}=2\pi$, and the solution of $|\alpha(\tau)|=\alpha_s$ is assumed to be $\theta_i(i=1,\cdots,M)$. $u_0,u_1,\cdots u_M$ are the signs of $\alpha(\tau)-\alpha_s$ within $[\theta_0\quad\theta_1],[\theta_1\quad\theta_2],\cdots,[\theta_M\quad\theta_{M+1}]$. The step function $H(u_m)$ is expressed as

$$H(u_m)=\begin{cases}1,(u_m\geq 0)\\0,(u_m<0)\end{cases},(m=0,1,\cdots,M) \tag{10}$$

In Eq. (9), $\operatorname{sgn}a$, $\operatorname{sgn}b$, and $\operatorname{sgn}c$ denote the relationship between $\alpha(\tau)$ and α_s in $[\theta_0\quad\theta_1],[\theta_1\quad\theta_2],\cdots,[\theta_M\quad\theta_{M+1}]$, respectively. They can be expressed as

$$\operatorname{sgn} a=\begin{cases}1,\alpha(\tau)>\alpha_s\\0,-\alpha_s\le\alpha(\tau)\le\alpha_s\\1,\alpha(\tau)<-\alpha_s\end{cases},\quad \operatorname{sgn} b=\begin{cases}-\alpha_s,\alpha(\tau)>\alpha_s\\0,-\alpha_s\le\alpha(\tau)\le\alpha_s\\\alpha_s,\alpha(\tau)<-\alpha_s\end{cases},\quad \operatorname{sgn} c=\begin{cases}-\alpha_s,\alpha(\tau)>\alpha_s\\0,-\alpha_s\le\alpha(\tau)\le\alpha_s\\\alpha_s,\alpha(\tau)<-\alpha_s\end{cases}\tag{11}$$

The above integral can be obtained directly by MATLAB software (MathWorks, USA). Here, the conversion of the nonlinear aeroelastic equation to the linear algebraic equations is completed. The solution of Eq. (8) is carried out by means of incremental steps and iterative steps, first prescribing an increment, such as $\Delta\omega$. According to Eq. (8), the increment $\Delta\mathbf{A}$ is obtained, and then the original $\mathbf{A}$ is replaced with $\mathbf{A}+\Delta\mathbf{A}$, and the new $\Delta\mathbf{A}$ is found, so the iterative loop is performed until $\mathbf{A}$ satisfies $\mathbf{R}=\mathbf{0}$. Next, a new increment $\Delta\omega$ is given, and $\omega_0+\Delta\omega$ is used to replace the original ω_0. The new ω_0 and $\mathbf{A}$ calculated by the last iteration are reused as the initial value in the harmonic balance calculation. After $\mathbf{A}$ is calculated for the new ω_0, the calculation proceeds to the next incremental procedure. Obviously, the incremental step and the iterative step are alternately cumbersome, and in the iterative calculation the selection of the initial value influences the iteration convergence. If the initial value is not appropriate, the result is not easy to converge, so here the iterative solution in Ref. [20] is used. For Eq. (8), the total number of unknowns is greater than the number of equations. When one of the elements in $\mathbf{A}$ is fixed, the corresponding increment is zero, and the number of unknowns in $\Delta\mathbf{A}$ is reduced by one, so that the total number of unknowns is equal to the number of equations; the solution is as follows:

$$\begin{bmatrix}K_{mc1,1} & K_{mc1,2} & \cdots & K_{mc1,j} & \cdots & K_{mc1,n}\\K_{mc2,1} & K_{mc22} & \cdots & K_{mc2,j} & \cdots & K_{mc2,n}\\\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\K_{mci,1} & K_{mci,2} & \cdots & K_{mci,j} & \cdots & K_{mci,n}\\\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\K_{mcn,1} & K_{mcn,2} & \cdots & K_{mcn,j} & \cdots & K_{mcn,n}\end{bmatrix}\begin{Bmatrix}\Delta a_{11}\\\Delta a_{12}\\\vdots\\\Delta a_{ij}=0\\\vdots\\\Delta b_{N,n}\end{Bmatrix}=\begin{Bmatrix}R_1\\R_2\\\vdots\\R_i\\\vdots\\R_n\end{Bmatrix}+\Delta\omega\begin{Bmatrix}R_{mc1}\\R_{mc2}\\\vdots\\R_{mci}\\\vdots\\R_{mcn}\end{Bmatrix}\tag{12}$$

Owing to $\Delta a_{ij}=0$, the above equation can be equivalently changed to achieve the following form:

$$\begin{bmatrix}K_{mc1,1} & K_{mc1,2} & \cdots & -R_{mc1} & \cdots & K_{mc1,n}\\K_{mc2,1} & K_{mc22} & \cdots & -R_{mc2} & \cdots & K_{mc2,n}\\\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\K_{mci,1} & K_{mci,2} & \cdots & -R_{mci} & \cdots & K_{mci,n}\\\vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\K_{mcn,1} & K_{mcn,2} & \cdots & -R_{mcn} & \cdots & K_{mcn,n}\end{bmatrix}\begin{Bmatrix}\Delta a_{11}\\\Delta a_{12}\\\vdots\\\Delta\omega\\\vdots\\\Delta b_{N,n}\end{Bmatrix}=\begin{Bmatrix}R_1\\R_2\\\vdots\\R_i\\\vdots\\R_n\end{Bmatrix}\tag{13}$$

$$\mathbf{K}_{mcr}\Delta\mathbf{A}_1=\mathbf{R}\tag{14}$$

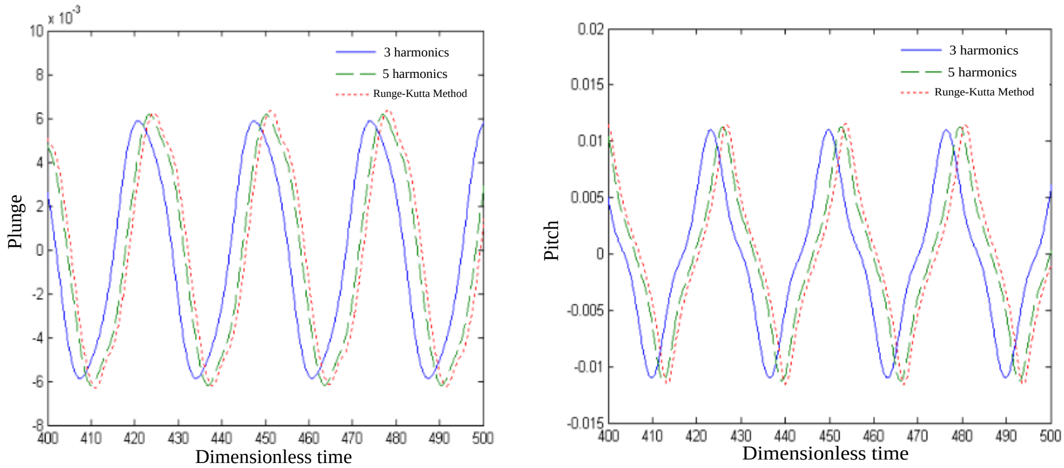
The Newton iterative method is used to solve Eq. (14). When $\Delta\mathbf{A}_1$ is obtained, $\mathbf{R}$ is the error vector, and $\mathbf{A}_1=\mathbf{A}+\Delta\mathbf{A}_1$, and $\omega_1=\omega_0+\Delta\omega$, which is substituted into $\mathbf{R}$ to determine whether its norm is zero. If the norm is greater than zero, then the iterative solution is continued until the norm is close to zero. At that moment, ω and $\mathbf{A}$ obtained are the coefficients and the coefficients corresponding to the harmonics of the aeroelastic system, respectively. The approximate solution of the system can also be obtained. The incremental harmonic balance method is completed to solve the aeroelastic system with piecewise structural nonlinearities.

4 Example Validations and Results Analysis

A two-dimensional airfoil is used, and the corresponding parameters are given in Ref. [19]. For the linear aeroelastic system (i.e., $\alpha_s=0$, $\eta_k=0$), the dimensionless linear flutter velocity obtained in Ref. [19] is $U_L=6.29$, which is a reference velocity in nonlinear aeroelastic analysis. First, the Runge-Kutta method is used to analyze the influence of the number of aerodynamic modes on the response curves of the plunge and pitch, which determines the number of aerodynamic modes retained. By comparison analysis, it is found that there are almost no differences between the response curves obtained by two reserved aerodynamic modes and those obtained by 10 reserved aerodynamic modes under the same velocity and initial conditions. Similar to the natural mode of structure, the contribution of high-order aerodynamic modes to system response is too small. Therefore, in order to save calculation time, two aerodynamic modes are reserved.

4.1 Example Validation

In Ref. [19], the numerical simulation is carried out in a relatively wide velocity range (i.e., $0 < U/U_L \le 0.8$). Owing to the damping effect of aerodynamic force, within $U/U_L < 0.143$, the plunge and pitch responses are damping stable motions and finally approach stable values. When U/U_L is close to 0.143, the two-dimensional airfoil exhibits limit cycle oscillation. Moreover, as the velocity continues to increase, the response tends to be complicated, e.g., period motion, quasi-periodic motion, and chaotic motion. The incremental harmonic balance method is mainly used to solve the periodic motion of nonlinear systems. Hence, in order to demonstrate that the incremental harmonic method can be used to solve the response of an aeroelastic system with piecewise structural nonlinearity, the plunge and pitch responses of the two-dimensional airfoil are obtained at the given velocity ($\eta_k = 3$, $\alpha_s = 0.5^\circ$).



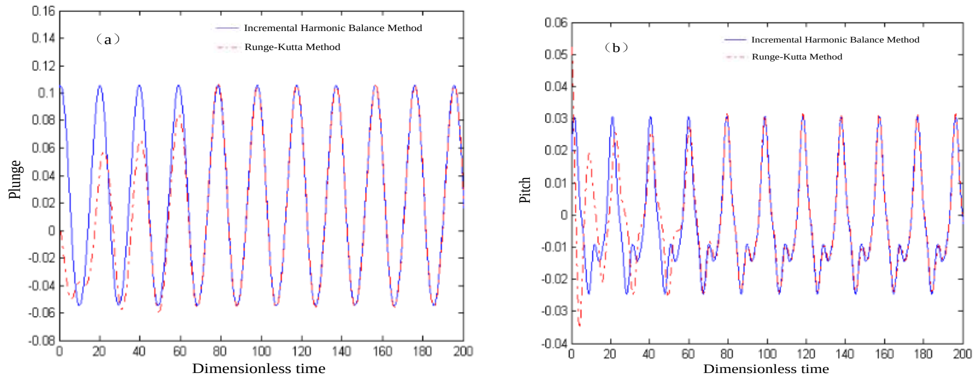
(a) (b)

Fig. 2 Plunge and pitch curves ($U/U_L = 0.2$)

Figure 2 shows the response using three and five harmonics involved in the calculation; that is, 1, $\cos\tau$, $\cos 2\tau$, $\cos 3\tau$, $\sin\tau$, $\sin 2\tau$, $\sin 3\tau$ and 1, $\cos\tau$, $\cos 2\tau$, $\cos 3\tau$, $\cos 4\tau$, $\cos 5\tau$, $\sin\tau$, $\sin 2\tau$, $\sin 3\tau$, $\sin 4\tau$, $\sin 5\tau$ for $U/U_L = 0.2$. It can be seen that as the number of harmonics increases, the plunge and pitch curves are closer to the numerical solution. However, as the number of harmonics increases, the order of Eq. (12) is higher. From the previous derivation, complex calculations such as integration and iteration are necessary when solving the nonlinear system by the incremental harmonic balance method. Higher-order equations are not conducive to solving the nonlinear aeroelastic system. However, the number of harmonics is too small, which will affect the accuracy of the solution.

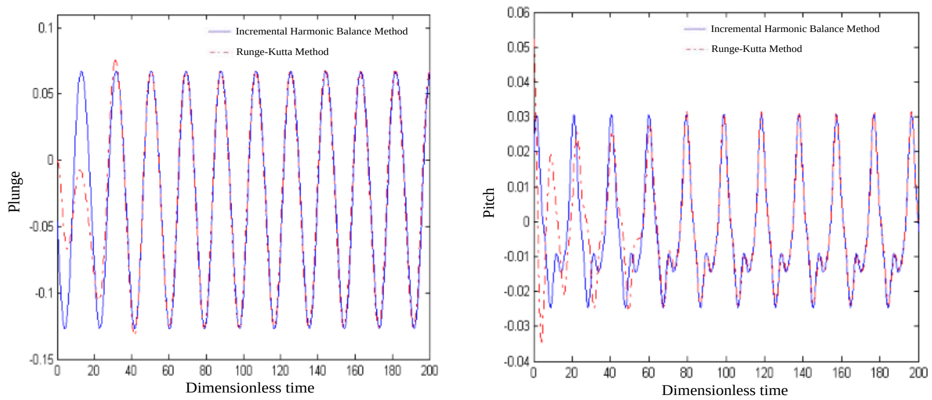
Obviously, there is a problem that the harmonic term and the harmonic frequency are uncertain when solving the periodic approximate solution of the nonlinear system by the incremental harmonic balance method. For nonlinear systems, different initial conditions mean a different combination of harmonics of the steady-state response of the system. It is necessary to properly select the number of harmonics and the corresponding frequency. The determination of the number of harmonics and frequency is also the key to the precision of the incremental harmonic balance method. In this study, the response of the system is obtained by a numerical method, and a fast Fourier transform is carried out to extract the dominant frequency components in the response, which determines the harmonic number and the frequency of the approximate solution. Then, the dominant frequency components are applied to the incremental harmonic balance method to solve the linearization equation, namely Eq. (8), which effectively determines the approximate solution of the nonlinear system and improves the solution precision.

For $U/U_L = 0.2$, the fast Fourier transform is applied to the numerical solution, implying that the plunge response consists mainly of 0.2284 and 0.7082 frequency components, and the pitch response is mainly composed of 0.2284, 0.7082, and 1.165 frequency components, in which 0.7082 and 1.165 are 3 and 5 times 0.2284, respectively. Therefore, although the three harmonics used are easy to use to solve the equations, which can obtain a lower-order linear algebraic equation, the real response of the system cannot be achieved. However, the five harmonics can cover all the dominant frequencies of the response, which can truly reflect the system response. Using a similar solving procedure, Figs. 3 to 5 show the plunge and pitch curves, respectively, for $U/U_L = 0.648,\ 0.7,\ \text{and}\ 0.8$.



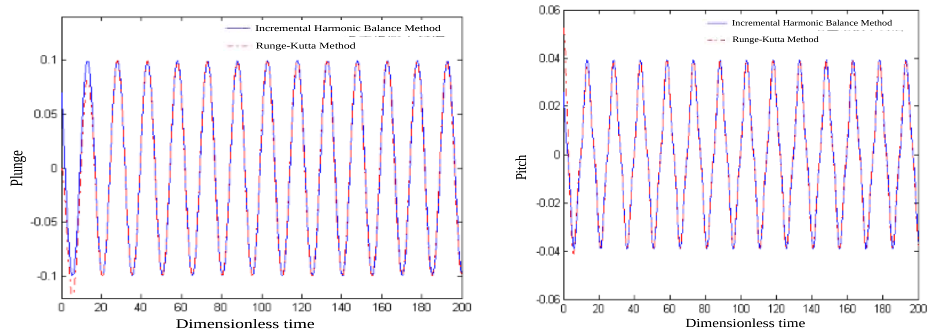
(a) (b)

Fig. 3 Plunge and pitch curves ($U/U_L = 0.648$)



(a) (b)

Fig. 4 Plunge and pitch curves ($U/U_L = 0.7$)



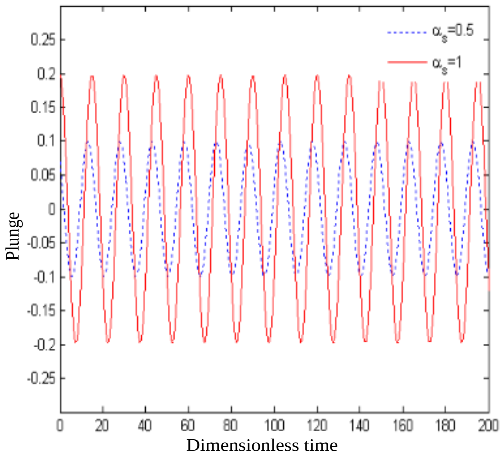
(a) (b)

Fig. 5 Plunge and pitch curves ($U/U_L = 0.8$)

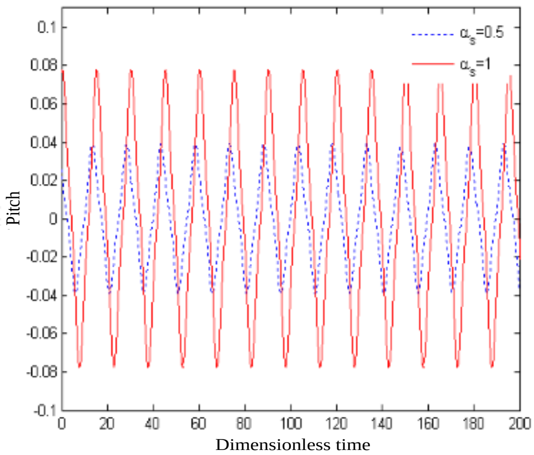
For $U/U_L = 0.648$, $U/U_L = 0.7$, and $U/U_L = 0.8$, the numerical solution is obtained by the Runge-Kutta method, and its dominant frequency is obtained by fast Fourier transform. Then, in accordance with the method proposed in this paper, the system response is obtained. Taking the response of $U/U_L = 0.8$ as an example, the plunge response consists mainly of a frequency of 0.4143, and the pitch response consists mainly of the frequency components of 0.4143 and 1.272, and the latter is approximately 3 times the former. Thus, three harmonics can be used to participate in the calculation; that is, 1, $\cos\tau$, $\cos 2\tau$, $\cos 3\tau$, $\sin\tau$, $\sin 2\tau$, $\sin 3\tau$. Similarly, for $U/U_L = 0.648$ and 0.7, four harmonics are used. Compared with the numerical results, it is found that the response is almost completely coincident with the increase of time, with the exception of the initial transient solution. This also validates the correctness of the proposed method, which uses the incremental harmonic balance method to solve the response of the piecewise nonlinear aeroelastic system. In addition, by making use of the fast Fourier transform of the numerical solution, the dominant frequency components of the response are obtained, which avoids the blind assumption of the periodic solution form of the nonlinear system, and improves the accuracy of the solution.

4.2 Parameter Analysis

In order to analyze the effect of free play α_s and stiffness ratio η_k on the response of nonlinear aeroelastic system, the response curves with different free plays and stiffness ratios are discussed. First, taking $\eta_k = 3$, $U/U_L = 0.8$, and different free play ($\alpha_s = 0.5^\circ$ and $\alpha_s = 1^\circ$), for example, the response curve is obtained by the proposed method. The comparison of the system responses with different free plays is shown in Fig. 6.



(a)

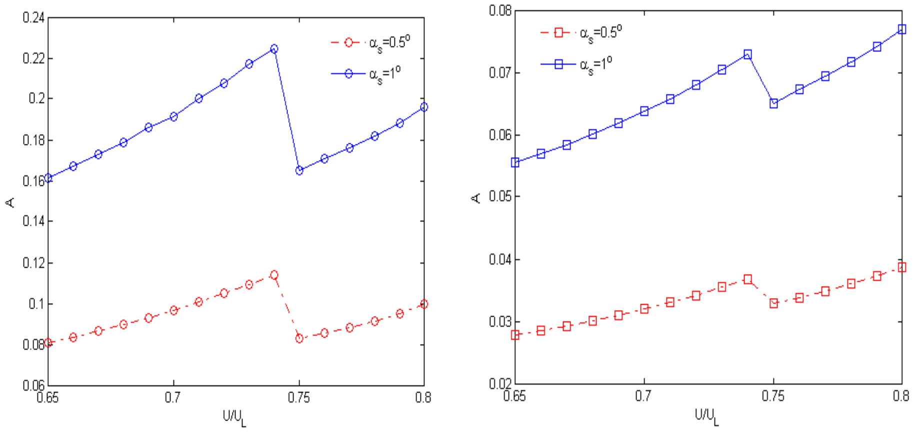


(b)

Fig. 6 Plunge and pitch curves ($U/U_L = 0.8$, $\eta_k = 3$)

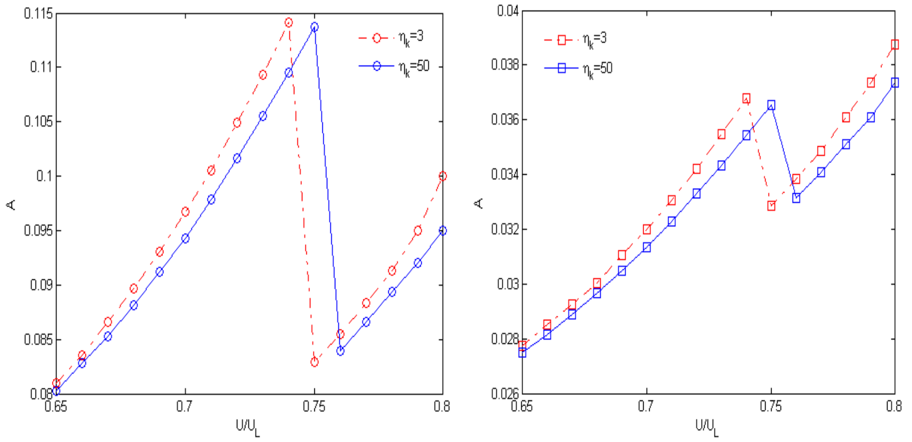
As can be seen from Fig. 6, with the increase in the free play, i.e., α_s, the oscillations of the curves intensify for the same velocity. When the free play equals 0.5, the maximum and minimum values of the plunge and pitch are Amax = 0.09872 and 0.03872 and Amin = −0.09805 and −0.03876, respectively. When the free play equals 1, the maximum and minimum value of the plunge and pitch are Amax = 0.1972 and 0.07688 and Amin = −0.1954 and −0.07671, respectively. Obviously, when the free play doubles, the maximums and minimums of the response are substantially twice the original response.

Now, the effect of system parameters on the response amplitude curve is investigated. It can be seen from the previous results that the response curve is not necessarily centrosymmetric, so it is defined as A = (Amax−Amin) / 2, and the response amplitude curves are shown in Figs. 7 and 8.



(a) (b)

Fig. 7 Amplitude curves with different free play α_s ($\eta_k = 3$)



(a) (b)

Fig. 8 Amplitude curves with different stiffness ratio η_k ($\alpha_s = 0.5^\circ$)

For $\eta_k = 3$, the greater free play means greater response amplitude with a given velocity ratio, and, in the case of $\alpha_s = 0.5^\circ$, the greater the stiffness ratio, the smaller the response amplitude for a given velocity ratio. For a given free play and stiffness ratio, in the case of $U/U_L \in [0.65,\ 0.8]$, as the velocity ratio increases, the response amplitude increases first and then decreases. Near $U/U_L = 0.75$, the response amplitude appears to jump. This phenomenon was also reported in Ref. [19], indicating the correctness of the proposed method.

5. Conclusions

In this paper, an approximate solution is proposed using the incremental harmonic balance method for an aeroelastic system with piecewise structural nonlinearity based on an unsteady vortex lattice model. The results are in good agreement with numerical results obtained by the Runge-Kutta method, and the influence of the parameters on the nonlinear aeroelasticity is also studied. The following conclusions can be drawn:

(1) For the processing of piecewise nonlinear terms, explicit expressions are worked out, which can provide ideas for other nonlinear processing. The application of the incremental harmonic balance method is extended.

(2) By the great number of harmonic terms, the approximate periodic solution of the nonlinear system can be obtained accurately. At the same time, the fast Fourier transform of the numerical solution is performed to determine the dominant frequency component in the response, which avoids the assumption of blindness of the system and improves the accuracy of the solution.

(3) For a given velocity ratio, the magnitude of the nonlinear system response can be reduced by

decreasing free play and increasing the stiffness ratio.

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